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A NEW METHOD OF MEASURING HUMIDITY IN A SMALL SPACE

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Abstract—A new method of measuring humidity in air has been developed using a small thermocouple as a probe. The junction of the thermocouple is cooled below the dewpoint of the air by passing a current through the thermocouple (Peltier cooling). The method was developed primarily for the measurement of vapour concentrations close to an evaporating surface in the so-called mass-transfer boundary layer. The spatial discrimination achieved is about 0·002 in.

NOMENCLATURE

- a, perimeter of thermocouple wire (equation 1) [cm];
- A, cross section area of thermocouple wire (equation 1) [cm^2];
- b, parameter expressing ratio of surface heat-transfer coefficient and wire perimeter to wire cross-section density specific heat (equation 1) [$s^{-\frac{1}{2}}$];
- E, surface heat-transfer coefficient of thermocouple wire (equation 1) [$W/degC/cm^2$];
- $f(x)$, temperature of a thermocouple wire at a distance x from the thermocouple junction at the instant of switching on the cooling current (equation 2) [°C];
- i, current in thermocouple (equation 1) [A];
- i_0, current in thermocouple which produces the identifiable signal referred to in the text (see Method of operation) [A];
- K, thermal conductivity of the thermocouple wire material (equation 2) [W/degC/cm];
- m, parameter expressing the ratio of the thermal conductivity to the product of the density and specific heat of the thermocouple wire material [$cm\,s^{-\frac{1}{2}}$];
- P, Peltier coefficient of the thermocouple junction (equation 2) [V];
- t, time (equation 1) [s];
- V_0, potential corresponding to a cooling current i_0 which produces identifiable signal (see text—Method of operation, also Fig. 6) [V];
- x, distance measured along thermocouple wires from thermocouple junction [cm].

Greek symbols

- α, dimensionless parameter expressing ratio of the product of the surface heat-transfer coefficient and the perimeter to the product of the thermal conductivity and the cross section area of the thermocouple wire (equation 2);
- η, parameter involving the parameter m and time (equation 2) [cm^{-1}];
- Δ, density of the thermocouple wire material (following equation 1) [$g\,cm^{-3}$];
- θ, temperature of the thermocouple wire (equation 1) [°C];
- $\theta(x, t)$, difference between the temperature of one of the thermocouple wires and the temperature of its surroundings a distance x from the junction and a

time t after the cooling current is started (equation 2) [degC];

λ, dimensionless number expressing the ratio in which heat absorbed at the thermocouple junction is conducted along the wires (following equation 1);

ρ, specific electrical resistance of thermo-thermocouple wire material (equation 1) [Ω cm];

σ, specific heat of thermocouple wire material (equation 1) [W s/g/degC].

Subscripts

1, 2, refer to the materials of the thermocouple wires, bismuth and busmuth–tin alloy.

INTRODUCTION

DUE TO the difficulty in making a small hygrometer, there have been no measurements of humidity gradients within the mass-transfer boundary layer in the same way that velocity and temperature gradients have been measured in the momentum- and heat-transfer boundary layers. Strunk [1] with a probe 1 mm in diameter showed that such a boundary layer exists, as large changes in vapour concentrations occur close to an evaporating surface. Thus in order to explore in detail the mass-transfer boundary layer, a hygrometer with a spacial discrimination of about 0·002 in was developed.

Alternative methods

A number of possible methods of measuring humidity in a small space were considered before the adopted method was conceived. These included methods of electrical conductivity; mechanical methods measuring the change of length of fibres; sampling; microwave refractometry and numerous other methods [2–4].

The electrical conductivity and mechanical methods suffered from the disadvantage that the accuracy and response decreased and hysteresis increased at high humidities. Microwave refraction techniques are unsuited to humidity measurements in a boundary layer. Sampling, or drawing off air through a fine tube and using a conventional hygrometer was discarded as being unreasonably slow and unsuitable in turbulent flow.

The Peltier Effect

The method adopted is based on the Peltier effect [5–7] which has of late found application in various thermo-electric cooling devices. In short, an electric current is passed through a junction between two thermo-electrically dissimilar metals. The effect of this current is to cool the junction. A thermocouple junction can function as a hygrometer by using the Peltier effect to cool the junction below the dewpoint temperature, whereupon the formation of moisture is detected, and the magnitude of the current at this point is a measure of the dewpoint temperature and hence the humidity.

The method is ideal for measurements in the boundary layer. The spacial discrimination is limited only by the size to which a single thermocouple junction can be made, and as the instrument is essentially a dewpoint hygrometer, it does not suffer from a loss of sensitivity at high humidities.

Early experiments

The first attempts to measure humidity using Peltier effect cooling were made with a cross of two thermo-electrically different metals, bismuth and an alloy of bismuth and 5% tin. The cross was resistance welded and soldered to stout copper leads. The circuit (Fig. 1) enabled a current to be passed through one leg of the cross, while the other leg acted as a thermocouple junction. The current was slowly increased and the potential across the thermocouple junction recorded. Plots of cooling current vs. thermocouple potential were straight lines but with a kink, thought to correspond to the dewpoint as moisture accumulated on the junction with cooling currents in excess of that required to give the kink. The method was slow and tedious, the kink ill-defined and did not

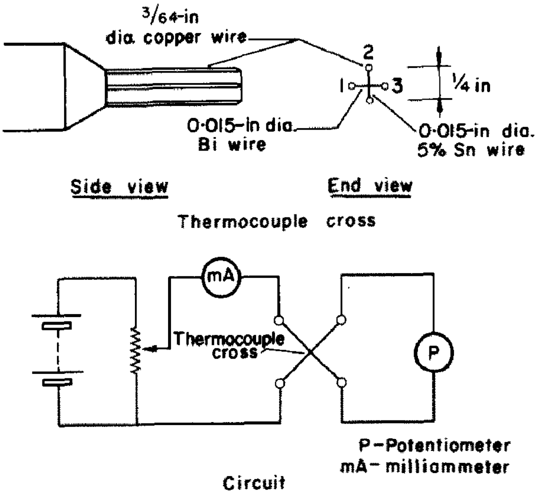

FIG. 1. Thermocouple cross and circuit. Bismuth and bismuth–tin alloy wires are soldered to copper supports and resistance welded together where they cross.

always appear. Thus the method was abandoned in favour of a single junction.

The first experiments with a single junction employed the technique described by Spanner [8] as the "ballistic technique". The circuit (Fig. 2) used a double throw–double pole switch to switch the thermocouple from a battery circuit cooling the junction to a galvanometer circuit which measured the temperature of the junction. The method was to cool the junction for a fixed time at a current known to be greater than that required to cool the junction below the dew-point and then to throw the switch, thus connecting the junction across the galvanometer and observing its maximum deflection. The method of Montieth and Owen [9] was also attempted. This is similar to the Spanner method except that a measure of the wet-bulb temperature is obtained by recording the galvanometer deflection at a fixed time after switching.

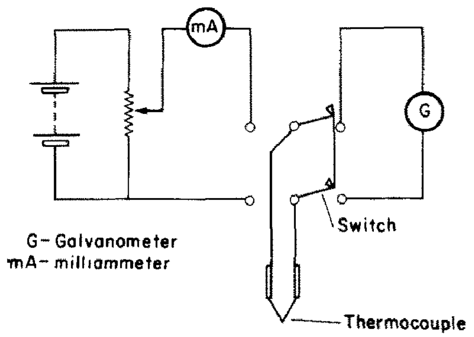

FIG. 2. Circuit using single thermocouple junction and double pole–double throw switch.

An attempt was made to calibrate this apparatus in moving air and after a number of modifications to the original circuit, including the use of rapid action switches and attempts to observe transient changes in the junction potential using an oscilloscope, a calibration was obtained. The calibration, however, exhibited a considerable scatter of results and provoked some thought as to how it could be improved.

The most serious limitations to the "ballistic method" seem to lie in the switching delay and the switching resistances which are difficult if not impossible to keep constant over long periods. Furthermore, the method depends on the ballistic throw of a galvanometer with a response time for full scale deflection of about two seconds giving a measure of the much more rapid evaporation from the junction.

These limitations were overcome by suitable electronic circuitry which alternately cooled and heated the junction. The junction was pulsed with the square wave form shown in Fig. 3.

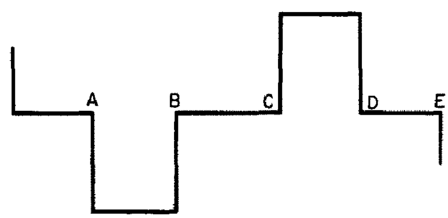

FIG. 3. Alternating square-wave current form.

The circuitry enables the thermocouple potential to be observed continually and the fast rise time of the square waves (about 20 μs) is several orders of magnitude faster than mechanical switching and is thought to be somewhat faster than the condensation of moisture onto a cold surface.

The method adopted

Wave-form generator. The wave form generator is shown in Fig. 4. It comprises four monostable multivibrators in cascade, each with independently variable period, provided by inserting shunts into a bank of capacitances, thus altering the period of the multivibrators. The period of the current waveform is 320 ms.

Two of the multivibrators drive a current reversing switch to produce the wave form of Fig. 3.

During A to B the junction is cooled.

B to C there is no applied current and the junction functions as a thermocouple.

C to D the junction is heated, evaporating condensation which may have occurred during A to B.

D to E is as for B to C.

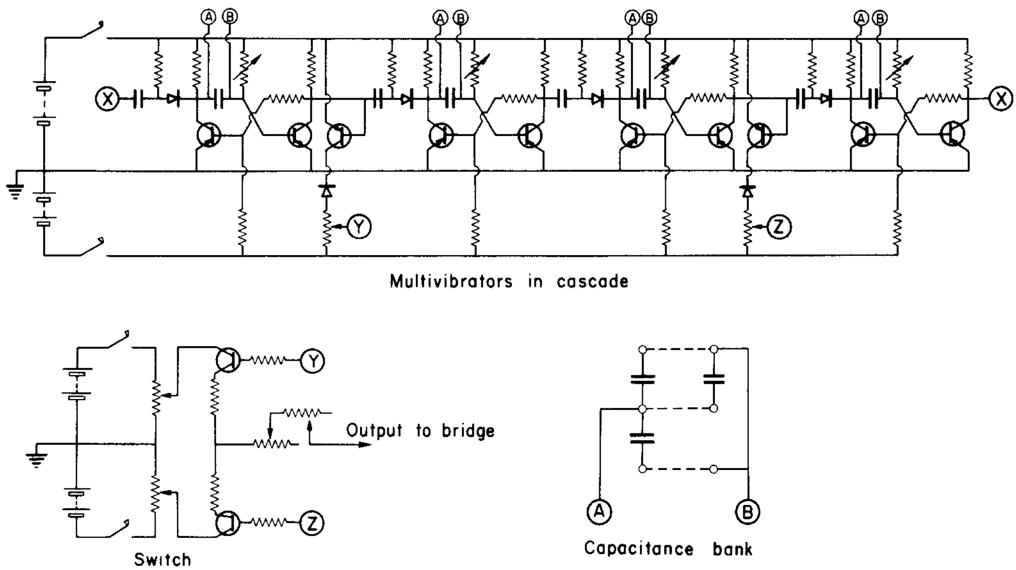

FIG. 4. Wave-form generator circuit.

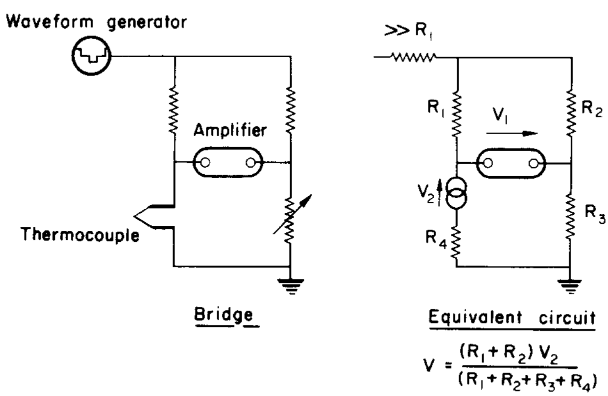

FIG. 5. Balanced bridge circuit.

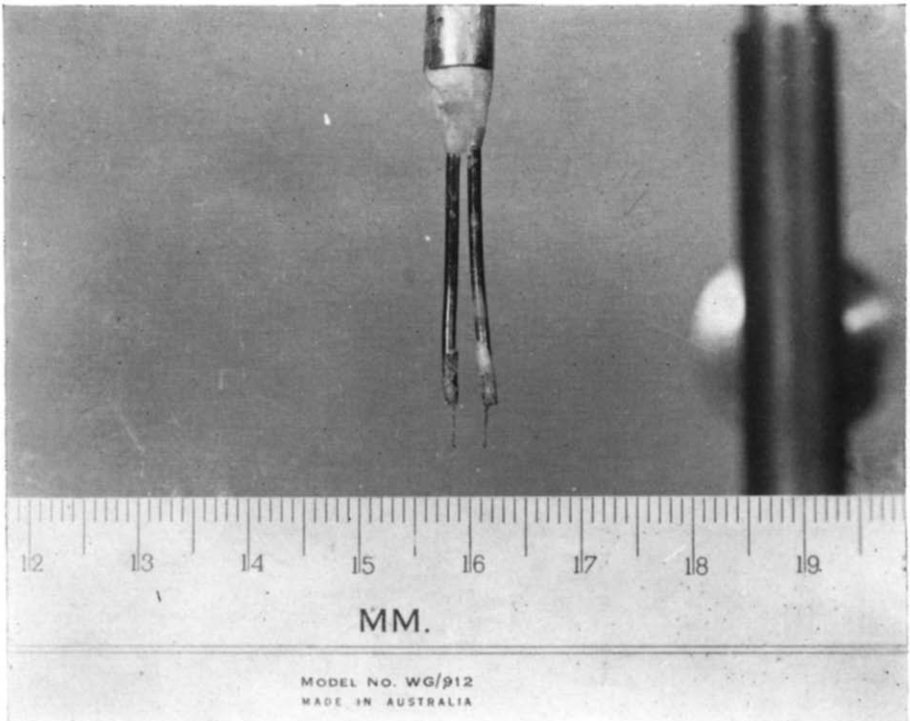

FIG. 6. Thermocouple junction used in calibration.

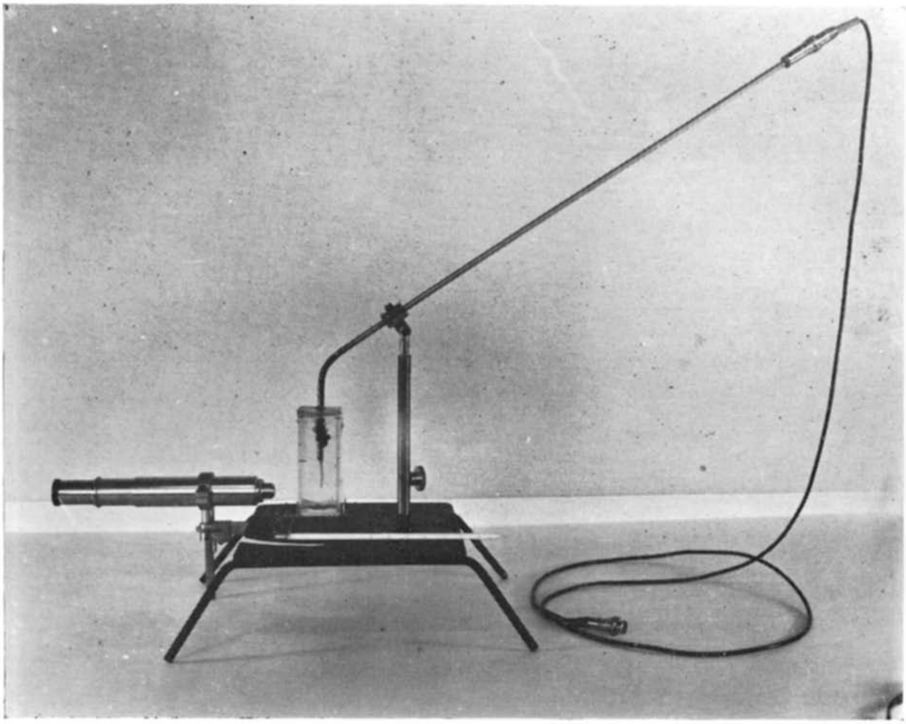

FIG. 9. Apparatus used in calibration in still air above saturated salt solutions.

At E the cycle repeats.

By balancing the amplitudes and durations of the heating and cooling currents it is possible to get no long-term heating or cooling of the junction. That is, the heat extracted from the junction during cooling is equal to the heat input to the junction during heating.

Bridge circuit. The junction is set up in a balanced bridge circuit (see Fig. 5). A sensitive differential amplifier and oscilloscope combination is used as the detecting instrument to observe the bridge unbalance.

Construction of thermocouple junction

Preparation of wires. The thermocouple wires are produced by the Strong [10] method. Briefly the method is to suck the molten metal into a glass tube, then draw out the tubing. The glass is etched from the wire with hydrofluoric acid solution. The author produced satisfactory wires (if rather brittle) by hand-drawing the glass over a Bunsen burner.

Manufacture of the junction. The wires are first soldered onto copper supports with Wood's metal and a flux of 40% $ZnCl_2$, 20% NH_4Cl, 40% H_2O. A soldering iron made from a fine needle with a Nichrome wire heating element was used. The work was carried out under a binocular microscope approximately 30 × magnification. The wires were crossed over and resistance welded together. Welding voltage needed to be carefully adjusted so as to fuse the wires without causing them to disintegrate. The junction was then carefully washed in a very mild caustic solution to neutralize the acidic flux. The author has produced temperatures as low as 12 degF below ambient with Bismuth, Bismuth–tin thermocouple wires about 0·0009–0·0012 in diameter arranged as shown in Fig. 6.

Method of operation. The method of operation is to increase the amplitude of the pulsing waveform until the signal detected by the amplifier and oscilloscope is of the identifiable form shown in Fig. 7. Also shown in Fig. 6 are the signals corresponding to a slight increase and a slight decrease in the amplitude of the pulsing waveform about the identifiable amplitude. The potential V_0 shown in Fig. 7 corresponding to a cooling current i_0 which produces the identifiable signal is recorded.

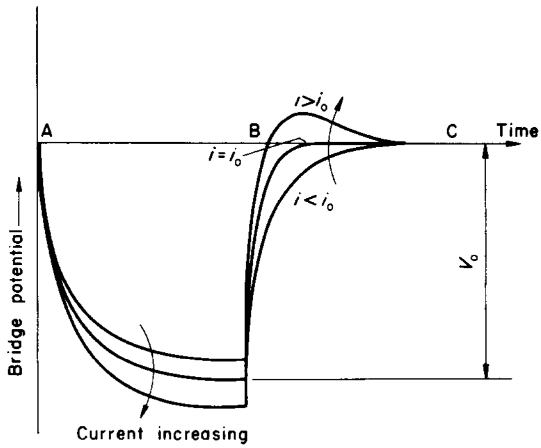

FIG. 7. Potential observed across bridge.

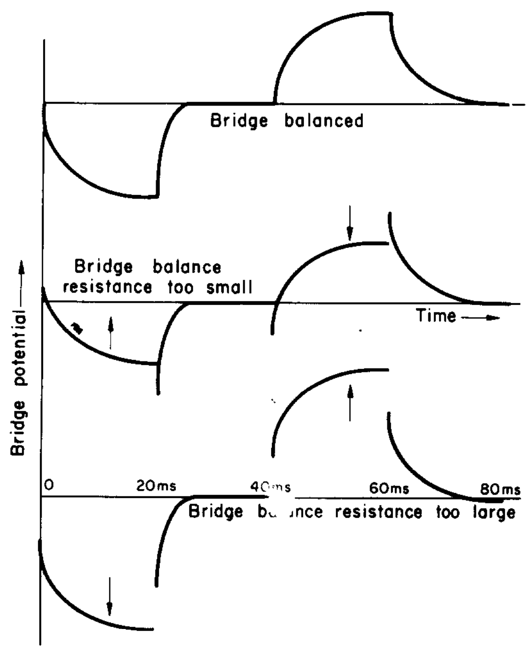

FIG. 8. Effect on observed potential of bridge unbalance.

The effect of unbalancing the bridge is that the signal from the thermocuple is superimposed on an attenuated pulsing waveform. This causes discontinuities in the observed signal as shown in Fig. 8. By adjusting the variable bridge resistance to eliminate the discontinuities the bridge may be balanced.

Principle of operation. In order to establish the principle of operation of the hygrometer and in particular to determine the cause of the observed change in the waveform, a mathematical model of the thermocouple junction was set up and an expression obtained for the temperature distribution in one of the thermocouple wires resulting from a cooling current in the thermocouple (see Appendix). The "humps" in the observed waveform (Fig. 7) are caused by the addition of exponential error function terms arising from the Peltier effect cooling and an exponential term arising from the Joule heating in the thermocouple wires.

A digital computer was used to calculate the behaviour of a thermocouple junction undergoing the steady pulsing current of Fig. 3. During the part of the cycle when there is no applied current, there is still a current flowing in the thermocouple due to the thermo-electric potential of the junction if the temperature of the junction is different from that of the rest of the circuit. This current is about one thousandth of the driving current used to cool the thermocouple, thus the Thompson effect [5] can be neglected as insignificant. In this case, the thermo-electric potential of the junction is directly proportional to the junction temperature and is independent of the temperature distribution along the wires. Thus by computing the temperature of the junction around the cycle it was possible to compute the junction potential observed by the amplifier and oscilloscope.

The computed signal closely resembled the observed potential change shown in Fig. 7 except in regions close to a change in current.

The same characteristic change in shape of the signal with a change in current as shown in Fig. 7 was produced by the calculation.

By varying the current, i, and the surface heat-transfer coefficient, it was possible to show that, for a particular value of the surface heat-transfer coefficient there corresponds only one value of the driving current i_0 which will give the easily identifiable horizontal signal. The surface heat-transfer coefficient, E, depends on whether or not the junction and wires are dry or wet, and if wet, depends on the amount of moisture condensed on the junction. By pulsing the junction long enough for steady-state conditions to establish, the quantity of water condensed each cycle will be constant and will depend on the humidity of the air surrounding the junction. Thus the potential V_0 (Fig. 7) recorded which is proportional to the driving current and hence to the surface heat-transfer coefficient serves as a measure of the humidity of the air at the junction. The identifiable horizontal signal does not correspond to the dewpoint or wet-bulb temperature since it indicates that there is enough moisture condensed on the junction to produce the effect.

Calibration. The thermocouple junction hygrometer was calibrated against the equilibrium humidity of still air above saturated salt solutions [11]. The apparatus is shown in Fig. 9. The Perspex box is hermatically sealed with petroleum jelly. The microscope is used for visual observations of condensation at low pulsing frequencies.

The calibration (Fig. 10) of potential corresponding to the identifiable horizontal signal against the depression of the dewpoint below ambient temperature is approximately linear. Above about 5 degF dewpoint depression, the hygrometer fails to register any further increase in dewpoint depression. This is because the maximum temperature drop obtainable with the bismuth, bismuth–tin thermocouple used is about 5 degF so that beyond this point, no moisture is condensed on the junction. It may be possible to increase the range of the instrument by using semiconductor materials, which have a much larger thermo-electric effect. (The Peltier coefficient of a n- and p-type bismuth–telluride

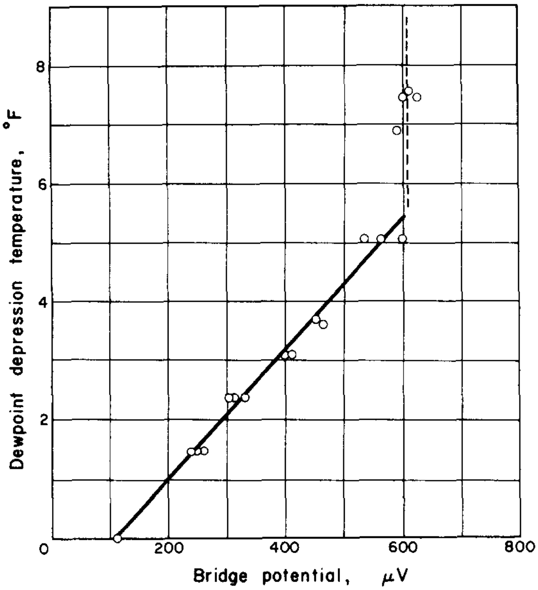

FIG. 10. Calibration of hygrometer in still air above saturated salt solutions.

couple at 0°C is 100–120 mV whereas that of a bismuth, bismuth–tin thermocouple at 0°C is 30 mV). Due to the long times necessary to ensure that the air above the saturated salt solution had reached equilibrium humidity, the calibration took place over a period of 28 days. This gives some idea of the long-term stability of the method.

CONCLUSION

It has been found possible to construct a small Peltier junction of bismuth, bismuth–tin wires of about 0·001-in diameter for the purpose of exploring humidity gradients close to an evaporating surface. The junction may be cooled as much as 5 degF below ambient temperature by passing a suitable current and water may be thus made to condense on it. Suitable electronic circuitry has been developed by means of which a steady voltage, related to the dewpoint depression below ambient temperature may be displayed.

It has been shown that the calibration is approximately linear in still air using saturated salt solutions as humidity standards.

At present, the author is calibrating the hygrometer in moving air, using a closed-circuit wind tunnel in which the air passes through a saturated salt solution. Following this calibration, it is intended to use the hygrometer for measuring the humidity gradient in a laminar boundary layer above an evaporating surface. It is hoped that, from measurements of mass-transfer boundary layers made possible by this hygrometer will come a better understanding of the mechanism of mass transfer, in the same way that measurements of momentum and thermal boundary layers have contributed to the understanding of momentum and heat-transfer.

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APPENDIX

Mathematical Model of Thermocouple

Theory

The differential equation of transient heat transfer from a wire, exchanging heat with its surroundings and conducting an electric current is [12]

$$\frac{\partial \theta}{\partial t} = m^2 \frac{\partial^2 \theta}{\partial x^2} - b^2\left(\theta + \frac{i^2 \rho}{EaA}\right) \tag{1}$$

where θ is temperature,
x is distance along the wire,
t is time,
$m = \sqrt{(K/\Delta\sigma)}$ where K is the thermal conductivity of the wire material,
Δ is the density of the wire material,
σ is the specific heat wire material,
$b = \sqrt{(Ea/A\Delta\sigma)}$ where E is the surface heat-transfer coefficient of the wire,
a is the wire perimeter,
A is the cross section area of the wire,
i is the current flowing in the wire,
and ρ is the specific electrical resistance of the wire material.

The solution to (1) applies to a thermocouple junction, carrying an electric current and subject to Peltier cooling at the junction, provided that the following boundary conditions are satisfied:

(a) $$\frac{\partial \theta}{\partial x} = -\lambda \frac{Pi}{KA} \quad \text{at } x = 0 \text{ independent of } t$$

where P is the Peltier coefficient of the junction

and $\lambda = \dfrac{\sqrt{(K_1\rho_1)}}{\sqrt{(K_1\rho_1)} + \sqrt{(K_2\rho_2)}}$, the subscripts referring to the thermocouple wires.

This condition provides for the Peltier effect at the thermocouple junction, with the heat absorbed at the junction being conducted along both wires in the ratio $\lambda:(1 - \lambda)$.

(b) $$\theta \to -\frac{i^2\rho}{EaA} \quad \text{as} \quad x, t \to \infty.$$

This condition states that, a long time after the current started, the wire temperature distant from the junction is as though the junction did not exist.

(c) $$\theta = f(x) \quad \text{at} \quad t = 0.$$

That is, there is a distribution $f(x)$ of temperature along the wire at the instant the cooling current is switched on.

The solution to (1) with these boundary conditions is

$$\theta(x, t) = \frac{\lambda Pi}{\sqrt{KAaE}}\{e^{-\alpha x}[1 - \mathrm{erf}(x\eta - b\sqrt{t})] - e^{\alpha x}[1 - \mathrm{erf}(x\eta + b\sqrt{t})]\}$$
$$- \frac{i^2\rho}{EaA}(1 - e^{-b^2 t}) + \frac{e^{-b^2 t}\eta}{\sqrt{\pi}}\left[\int_0^\infty f(\xi)\, e^{-(\xi - x)^2 \eta^2}\, d\xi\right] \tag{2}$$

where

$$\eta = \frac{1}{2m\sqrt{t}}$$

$$\alpha = \sqrt{(Ea/KA)}$$

$$\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-s^2}\,\mathrm{d}s \qquad \text{(the error function)}$$

and $\theta(x, t)$ is the difference between the temperature of one of the wires and the temperature of its surroundings a distance x from the junction and a time t after the current is started.

This solution was obtained by combining solutions of equation (1) [8, 12, 13] which have different boundary conditions but which combine to give the required boundary conditions. The temperature distribution in the other thermocouple wire may be obtained by substituting $(1 - \lambda)$ for λ in (2) and using the values of E, K, ρ, Δ and σ relevant to that wire.

Résumé—On a mis en oeuvre une nouvelle méthode de mesure de l'humidité de l'air en employant comme sonde un petit thermocouple. La jonction du thermocouple est refroidie au-dessous du point de rosée de l'air en faisant passer un courant électrique à travers le thermocouple (refroidissement par effet Peltier). La méthode a été exploitée d'abord pour la mesure des concentrations de vapeur au voisinage d'une surface évaporante dans la couche limite "de transport de masse". La résolution spatiale est d'environ 50 microns.

Zusammenfassung—Eine neue Methode zur Messung der Luftfeuchtigkeit mit Hilfe kleiner Thermoelemente wurde entwickelt. Die Lötstelle des Thermoelements wird unter den Taupunkt der Luft abgekühlt, indem man Strom durch das Element fliessen lässt (Peltierkühlung). Die Methode wurde primär entwickelt zur Messung der Dampfkonzentrationen über einer verdunstenden Oberfläche in der sogenannten Stoffübergangsgrenzschicht. Die räumliche Auflösung erreichte etwa 0,05 mm.

Аннотация—Разработан новый метод измерения влажности воздуха с помощью термопары в качестве зонда. Спай термопары охлаждался ниже точки росы для воздуха электрическим током, пропускаемым через термопару (эффект Пельтье). Метод первоначально предназначался для измерения концентрации пара вблизи поверхности испарения в так называемом массообменном пограничном слое.

Разрешающая способность датчика составляла 0,002 дюйма.

x